

## Teaching number fact and computational fluency: teachers' perceptions and impact upon practice.

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This paper presents primary school teachers' perceptions associated with their engagement in a project aimed at building Year 3 and Year 4 students' number fact and computational fluency. Throughout the project, teachers were provided with classroom ideas and suggestions for engaging students in number fact practice and for generating student discussions about computational strategies. They were also presented ideas for a teaching sequence for building number facts through thinking strategies; that is, a strategies approach. The impact of this program, through survey responses reveal marked shifts (pre- to post-project) in teacher attitudes, skills, knowledge and practices around teaching fluency with number facts. The impact of the teacher implementing a strategies approach to teaching number facts and computational fluency is discussed.

### Introduction

This paper reports an aspect of a larger research study that involved primary school teachers ( $n = 41$ ), Year Three students ( $n = 460$ ) and Year Four students ( $n = 472$ ) from five primary schools. The focus of the larger research project was the students' development of basic number fact recall and mental computation (fluency) by immersing them in an enriched number facts strategies approach program facilitated by their classroom teacher. In particular, the study focused on addition and subtraction number fact fluency (Year Three), multiplication and division number fact fluency (Year Four) and student dispositions towards mathematics and number facts. A survey was provided to the teachers, predominantly to inform program improvements, but also to determine project impact and draw implications for project sustainability and scalability. An analysis of the teachers' response to the project is the focus of this paper.

In this project, all Year Three and Year Four teachers from five primary schools committed to allocating one hour per week to the development of number facts as part of their regular mathematics program (typically equivalent to 3 x 20 minute sessions) over an intervention period of approximately 20 weeks (two school terms). Teachers were provided with suggestions for classroom activities and also presented with a suggested sequence for teaching addition and multiplication facts through the use of strategies; that is a strategies approach (McIntosh & Dole, 2000).

Prior to and at the end of the teaching period, students completed a Number Facts Quiz. Table 1 provides the pre- and post-test mean scores for the Year three students associated with addition and subtraction facts, and for the Year Four students associated with multiplication and division facts, as well as the relative growth.

Year	Pre-test mean score (%)	Post-test mean score (%)	Relative growth (%)
3	53	81	53
4	27	59	119

*Table 1:* Pre- and post-test mean scores and relative growth for each year level.

From Table 1, it can be seen that the relative growth for both Year Three and Four students is considerable. The relative growth for the Year Four students on multiplication and division facts is more than double the relative growth for Year Three students on addition and subtraction facts (Full details of this study are forthcoming).

### *Conceptual Framework*

Current research continues to highlight the importance of the teacher in developing fluency in number and computation and how games, strategies and number talks can assist (e.g., Boaler 2015; Ferguson, 2015; Gore, 2016). The conceptual framework of the strategies approach to number facts is located in more historical and established research. Historic research has shown that explicit teaching of strategies for particular groups of basic facts can facilitate fact recall and application in problem solving (e.g., Mercer & Miller, 1992; Rightsel & Thornton, 1985; Steinberg, 1985; Thornton & Smith, 1988). Children also use known facts (i.e., those that are automatic and can be retrieved instantly) to derive other facts (Hierdsfield, 1999). Sequencing instruction based on strategies that relate to particular groups of facts has been the basis of previous research (e.g., Rightsel & Thornton, 1985; Steinberg, 1985; Thornton & Smith, 1988). The design of instruction based on targeting facts grouped according to strategy type and consolidating particular groups of facts to enable students to apply derived fact strategies, is the conceptual basis upon which the strategies approach, presented to teachers in this project, was designed.

Achieving number facts fluency with a strong foundation of number sense “grows out of discovering the numerous patterns and relationship that interconnect the basic combination” (Baroody, 2006, p. 24). Teachers have been exploring these practices to promote fluency and have discovered the power of number talks and games (Buchholz, 2004; Chesney, 2013; Ferguson, 2015; Gore, 2016). Teaching and learning that focus on additive and multiplicative thinking, encourage rich number discussions and engage with number sense in a non-fearful environment have had a profound effect on students’ automaticity (Boaler, 2015; Godfrey & Stone, 2013). Teacher practices that promote discussion of patterns, relationships and connections outweigh the blind memorisation of number facts. Teachers adopting a strategies approach to teaching recall of basic facts as well as a wide range of thinking strategies, places students in a strong position to develop fluency in mental computation.

### The Study

The study adopted a quasi-experimental design and included a research and development focus, with the intervention developed in collaboration with project teachers following principles of research-based professional development proposed by Loucks-Horsley, Love, Stiles, Mundry and Hewson, (2009). The importance of providing teachers with the freedom to design and develop their own lesson sequences and approaches for their own students and to make decisions about how to implement ideas presented to them during the professional development workshops was recognised.

A key element of the project design was to align the project goals with the needs of participating teachers. The project team recognised the importance of providing teachers

with authentic, practice-based learning opportunities drawn from research into basic number fact development, opportunities to experience these investigations as learners themselves, and opportunities to share their ideas and experiences with colleagues, including the challenges encountered and their insights into the process. The importance of providing teachers with opportunities to design and develop their own lesson sequences and approaches for their own students was also recognised. Teachers must be provided with sufficient time to design, trial and evaluate their classroom approaches. This serves to validate teachers' practice and provide ongoing support and encouragement (Goos, Geiger & Dole, 2014; Hilton, Hilton, Dole & Goos, 2016). Throughout this project, there was continuous involvement with all teachers through school visits, meetings with school-based lead teachers and informal and ad hoc classroom observations undertaken members of the research team. Whilst the researchers provided input through school meetings with project teachers at the commencement and mid-way through the project, teachers adapted and planned number fact experiences to suit their own teaching style and that would align their own classroom culture.

For the aspect of the study reported here, the analysis of data was underpinned by the following research question:

- To what extent are teachers' attitudes, skills, knowledge and practices associated with teaching and learning number fact in Year Three and Year Four classrooms impacted by engagement in an extended research project?
- What are teachers' perceptions of teaching number facts through a strategies approach?

### *Instrument*

An online survey comprising of 13 Likert agree/disagree scale questions and one open response question was designed by the researchers. The purpose of the *Teacher Survey* was to determine teachers' level of agreement towards the project and its impact on their teaching practice, skills, knowledge and attitudes. The survey was designed to be administered to all teachers at the end of the project. The design of the survey was a two-tiered approach, with teachers being provided with a statement and then a two-tiered scale for them to consider their response at two different junctures: at the beginning of the project and at the end. This instrument follows a similar design to one developed by researcher Dole in previous work (see Dole & Beswick, 2002) and administered online to preserve data accuracy (Jorgensen et al., 1998; Smyth et al., 1997). The survey presented teachers with a statement and required them to indicate their level of agreement ranging from strongly agree to strongly disagree. Items included such questions as *I enjoy teaching number facts to my students; I have the pedagogical content knowledge to teach number fact fluency to my students.*

### *Procedure*

At the beginning of the school year, project teachers from each of the five schools engaged in a half-day professional development session with one of the project leaders (Carmichael). This occurred at each school, with Year Three and Year Four teachers released from their classrooms for half a day each. The strategies approach to teaching basic fact recall and mental computation were presented. The place of mental computation and basic fact recall within the Australian Curriculum: Mathematics was also revisited. During this professional development teachers were also given an opportunity to share resources and discuss their attitudes, experiences and beliefs about teaching number facts.

The *Teacher Survey* was implemented at the final stage of the research project and teachers had a two-week timeframe to complete the online survey. All project teachers were

informed and gave online consent for the collection of survey data. Consent was based on agreement that the online survey data was anonymous, and the participants were free to withdraw from the survey at any time without prejudice.

## Results

The *Teacher Survey* was analysed using descriptive statistical methods interpreted by analysis of the scores on each item of the survey. Table 2 presents project teachers' responses to each of the survey items. Teachers were required to indicate their level of agreement with the item presented at two different time points: at the beginning of the project and at the end of the project. They were provided with a two-tiered Likert scale to make a comparison of their opinions. Twenty-five project teachers completed the survey, which is an approximate 60% return rate.

Table 2

*Teacher level of agreement (%) about the teaching and learning of number facts pre- and post the project (SA strong agree, A agree, N neutral, D disagree, SD strongly disagree)(n=25)*

Survey Item		SD	D	N	A	SA
1. Fluency with number facts requires explicit teaching of thinking strategies.	Pre	0	9	4	44	44
	Post	5	0	0	19	76
2. I have the pedagogical content knowledge to teach number fact fluency to my students.	Pre	0	15	15	45	25
	Post	0	5	5	30	60
3. I am familiar with the sequence for introducing number facts thinking strategies.	Pre	0	38	14	43	5
	Post	0	0	0	64	36
4. I have trust in the number sense approach to teaching number facts.	Pre	0	14	36	46	5
	Post	0	0	0	50	50
5. Student self-monitoring leads to improved student outcomes.	Pre	0	5	23	50	23
	Post	0	0	5	33	62
6. I am confident in delivering a number talk session.	Pre	10	25	25	30	10
	Post	0	0	5	58	37
7. I enjoy teaching number facts to my students.	Pre	0	14	10	48	29
	Post	0	0	0	10	90
8. Routine practice improves my students' number fact knowledge.	Pre	0	5	0	48	48
	Post	0	0	0	14	86
9. The language used and approach to teaching number facts is consistent in my class.	Pre	0	5	23	50	23
	Post	0	0	0	33	67
10. My approach to teaching number facts is consistent with other teachers in this school.	Pre	0	17	30	52	0
	Post	0	5	0	62	33
11. I am committed to continuing the teaching practices of this project beyond 2017.	Pre					
	Post	0	0	0	8	92
12. Time allocated to number facts teaching and learning in this project (60 minutes/week) was manageable.	Pre					
	Post	4	4	4	52	36
13. I have been well-supported to implement this project at my school.	Pre					
	Post	0	0	0	44	56

To facilitate a discussion of results, data from the teacher survey are described at pre- and post-survey time junctures, even though teachers completed the survey at the same time. Teacher survey data show marked shifts in opinions as a result of the project. At the beginning of the project, teachers indicated general agreement with many items with often a similar percentage of teachers indicating that they agree or strongly agree with the statement presented. In the post-survey, there is a much stronger and more definite agreement, with many teachers indicating that they strongly agree with particular statements.

### *Teacher attitudes and views*

Prior to the project, 48% and 29% of teachers agreed or strongly agreed respectively that they enjoyed teaching number facts to their students, with 10% neutral and 14% disagreeing. At the post-survey stage, 81% of teachers strongly agreed that they enjoyed teaching number facts to their students, and 0% of neutral or disagreement. This shift in opinion provides a strong indication with which the teachers valued this project. One teacher attributes the enjoyment to the level of support given; "I have enjoyed the support the project has given me...it has been valuable to me". Teacher confidence in teaching number facts also shifted. In the pre-survey, only 40% of teachers agreed that they were confident. At the post-survey stage, this changed to 95% agreement in confidence with only 5% neutral. One teacher reflects on their confidence level by stating; "I have enjoyed the program and watching mine and the students' confidence grow in number sense". The project also appears to have impacted teachers' views about the explicit teaching of thinking strategies for number facts. At the pre-survey stage, 44% of teachers both agreed and strongly agreed respectively with this statement. At the post-survey stage, 76% of teachers strongly agreed with this statement, with 19% indicating agreement. Interestingly, 5% of teachers indicated that they strongly disagreed with this statement at the post-survey stage of the project, which is a change from 0% who strongly disagreed with this statement at the pre-survey stage of discussion of results. This suggests that, prior to this project, some teachers held a relatively strong view that number facts should not be the focus of explicit teaching. It could be assumed, therefore, that there would be no planned focus for teaching of strategies for learning number facts in their classrooms. Four percent of teachers indicated that they were neutral about the explicit teaching of thinking strategies for number facts prior to the project, which possibly suggests that their approach to number facts was not overly focused in their classrooms. At the end of the project, there is some polarisation of views about teaching thinking strategies for number facts. Overwhelmingly, 95% of teachers indicated that teaching number facts through strategies was extremely important, with 5% of teachers indicating that they strongly disagreed that fluency with number facts requires explicit teaching of thinking strategies. No teachers indicated that they disagreed, or were neutral about this. Further investigation into the approach to developing numbers facts taken in teachers' classrooms who strongly disagreed with this statement would assist in interpreting the pattern in this data.

### *Teacher practices, skills and knowledge*

One of the other factors behind a successful numeracy program, is the competence of the teacher (Brown, Askew, Rhodes, Denvir, Ranson & William, 2001). The data suggest significant growth in the teachers' pedagogical competence. It is clear that the project impacted teacher knowledge about sequencing instruction for number fact development. At the pre-survey stage, 38% of teachers stated that they did not agree that they were familiar with a sequence for introducing number fact thinking strategies, with 14% stating that they were neutral about this statement. In the post-survey, there was 100% agreement that teachers were familiar with such a sequence, with 64% and 36% respectively agreeing or

strongly agreeing on this. Further acknowledgement about the value of the suggested approach for teaching number facts is from teachers' responses when asked if they had trust in it. Responses on the pre-survey show a range of opinions, with 14% disagreeing, 36% neutral, 46% agreeing and only 5% strongly agreeing. On the post-survey, responses of 50% both agreeing and strongly agreeing were returned. Being part of this project built teachers' trust in this approach. There was also an indication that teachers used consistent language and approach in their classrooms in relation to number fact teaching. Prior to the project, teachers' responses to this item returned responses ranging from disagree (5%), neutral (23%), agree (50%) and strongly agree (23%). On the post-survey, there was 100% agreement that this was the case (33% agreement; 67% strong agreement).

A similarly high shift occurred in response to the statement that '*routine practice improves students' number fact knowledge*'. At the pre-survey stage, only 48% of teachers agreed and 48% strongly agreed with this statement; at the end of the project, 86% of teachers indicated that they strongly agreed with this statement and 14% agreed. The strength of agreement with this statement indicates the impact that this project has on teachers' practice.

One of the features of this approach to assisting students develop number facts is the inclusion of encouraging students to self-monitor their number fact progress. In the pre-survey, teachers were asked to comment on whether they believed that student self-monitoring lead to improved student outcomes. Responses to this item were mixed, with no-one strongly disagreeing with this statement (23% of teachers neutral, 50% agreeing and 23% strongly agreeing). At the post-survey stage, there was a strong shift in agreement, with 95% of teachers agreeing with this statement (62% strong agreement; 33% agreement) and only 5% neutral.

The project also appears to have had an impact on consistency of approach to teaching number facts at the whole school level. At the pre-survey stage, only 52% of teachers indicated that this was the case, with 30% neutral and 17% disagreeing. At the post-survey, there was 95% agreement that there was a consistent approach at their school. There was a 5% response that this was not the case.

The last three Likert survey questions were not on the pre-survey and all three questions provided valuable information about the future of the project, the time allocation, and their feelings of support whilst participating in the project. There was 100% commitment to continuing to implement the number fact teaching strategies that they had employed throughout the project and 100% agreement on feelings of support to implement the project this year. There was 95% agreement that the time allocation to teaching number facts, as advocated throughout this project, was manageable as part of the mathematics program. One teacher commented; "A great project well worth the time and I have learnt so much! Thanks".

## Discussion

The value of the enriched strategies approach teaching agenda for number facts is highlighted through teacher survey responses prior to the commencement of the project and at the end of the project. As seen in the teacher survey data, there was a range of responses to the survey item asking teachers whether they enjoyed teaching number facts to their students. At the pre-survey point, 14% of teachers indicated that they did not enjoy teaching number facts to their students, and 10% expressed indifference. At the post-survey point, there was 100% agreement (81% strongly agreed; 9% agreed) that project teachers enjoyed teaching number facts. This provides further evidence of the value to teachers of this explicit teaching agenda for number facts.

Of importance in this project was to empower teachers to build confidence in teaching number facts; to provide teachers with opportunities to learn practices, skills and knowledge that supported number facts fluency; and to assist teachers in embedding a number facts approach within their daily mathematics program after the project had ended. Teacher survey responses provided strong evidence that this project met these objectives. In the post-survey at the conclusion of the project, teachers were asked to indicate the level of agreement they had about the following statement: 'I am committed to continuing the teaching practices in this project beyond 2017'. Of the 25 teachers who completed the post-survey, there was 100% agreement with this statement, of which 92% indicated that they strongly agreed to do so. This is a very positive response, indicating the extent to which this project made an impact on teachers' practice. This provides further evidence of the power of the research methodology of this project that aligned with the Louck-Horsley et al. (2009) approach to professional development. In this project, a team approach was taken and teachers and school leaders were regarded as partners in the project. All participating schools included all Year Three and Year Four teachers and a strong partnership at each school level was built. An iterative cycle of action occurred as teachers tried activities in their classrooms and shared these with each other through interactions with each other and through the school leaders. Survey responses also indicated that 100% of teacher participants felt that they had been well-supported to implement this project at their school. Teachers in this project were linked with sources of knowledge and stimulation from outside their schools through having the region's numeracy coach visiting the schools throughout the project. Researchers from the university also were part of this project and provided further sources of knowledge. Evidence that practices that teachers employed during this project will continue and that an enriched approach to number facts will be sustained in the Year Three and Year Four classrooms comes from teacher survey responses to the following item: Time allocation to number facts teaching and learning in this project (60 minutes per week) is manageable. Whilst there was not 100% agreement with this statement, there was 88% agreement (36% strongly agreed), with 4% each stating neutral, disagree or strongly disagree. For the majority of teachers in this project, it is reasonable to suggest that their knowledge of teaching number facts was greatly enhanced as a result of this project and that the number facts approach would continue to be incorporated into their mathematics teaching.

### Concluding comments

As a result of analysing the data, the power of teachers engaging in a strategies approach to teaching number facts and computational fluency is evident. The results of this study suggest the value of targeting groups of basic facts via a strategies approach to facilitate recall and application in problem solving (e.g., Mercer & Miller, 1992; Rightsel & Thornton, 1985; Steinberg, 1985; Thornton & Smith, 1988). This approach not only supports students but also teachers' practice, skills, knowledge and attitudes. Teachers' strong shift in agreement with survey items associated with targeted teaching of number facts provides clear evidence of how this project provided clarity and direction to teachers for teaching number facts. Because of the fundamental importance of number fact recall for computation fluency, this research highlights that continued attention to this aspect of the curriculum is warranted.

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